# Polarisation and entanglement of photons 09/05/2020

## A new non-quantum approach

Pascal DUBOIS

**Key words**: photon; polarisation; model; entanglement; quantum; entangled photons; theorem; Bell inequalities; wave theory.

In the note entitled "Gravitational field, Fundamental Principle of Dynamics and Quantum Mechanics"[1], we showed that the concept of wave-particle duality could be expressed in a classical way by introducing a pilot wave that has a physical reality. This wave is attached to the gravitational field and obeys an equation close to the Schrödinger equation.

Another basic concept of quantum mechanics is entanglement. For a pair of particles that are entangled with respect to a given property (e.g. photon polarisation), the corresponding state vector is non-separable: it cannot be factorised into a tensor product of two vectors, each defining the state of a single particle. As a result, whatever the distance separating two entangled particles, a measurement made on one particle influences the other particle.

The non-local nature of quantum mechanics has given rise to much discussion[2] . Following repeated experiments[3] , quantum entanglement is accepted as a physical reality, although no explanation has been proposed other than the possibility of the existence of non-separable states, which is not prohibited by the quantum formalism.

In this note, we show that a new approach to the notion of photon polarisation makes it possible to recover the basic results of quantum mechanics. The probabilization of the results does not reflect a fundamental indeterminacy inherent in the photon, but comes from taking into account a polarization model that introduces a dispersion around a principal direction. The polarisation state of the photons is not modified by the polariser, but the latter selects a cohort of photos whose distribution depends on the orientation of the polariser.

Although this approach cannot be described as non-local, it allows us to recover the results of experiments carried out on entangled photons. We explain why there is no contradiction with Bell's theorem.

Finally, the proposed model for photon polarisation makes it possible to establish a link with classical wave theory, which has yet to be developed.

[1] https://www.aimer-la-physique.com

[2] including those linked to the EPR paradox (introduced by Einstein, Podolsky and Rosen in 1935).

[3] and in particular those of Alain Aspect on pairs of photons entangled in polarisation (1980-1982)

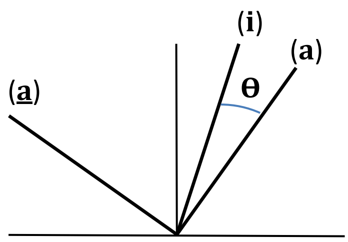
## 1. *What is polarisation?*

### 1.1. Classical wave theory

The polarisation of an electromagnetic wave describes the way in which the electric and magnetic fields oscillate. For a monochromatic plane wave, this oscillation, which occurs in the wave plane perpendicular to the direction of propagation, can be rectilinear, circular or elliptical (by linear superposition of two rectilinear polarisation states).

A natural light source does not emit a polarised beam. However, a polarised beam can be obtained by using a polarising filter, which allows only the fraction of the beam with the chosen state of polarisation to pass through. For example, a straight polarised beam can be obtained by interposing a polaroid, which only transmits vibrations parallel to a given direction.

By using two separate polarisers or a special device such as a birefringent plate, a linearly polarised light beam can be split into two orthogonal directions.




The two directions are (**a**) and ($\underline{\mathbf{a}}$) and the direction of polarisation of the incident beam is (**i**).
Let Θ be the angle (**i**, **a**).

If the intensity of the incident beam is I, the intensity exiting through channel **a** is:

$$I_a = I \cos^2 \Theta \qquad (1)$$

And the output from the channel $\underline{\mathbf{a}}$ is :

$$I_{\underline{a}} = I \sin^2 \Theta \qquad (2)$$

These expressions constitute Malus' law.

If a single-channel polariser is used, part of the intensity is transmitted and the rest is absorbed by the device. If the beam is unpolarised (it is assumed that all polarisation directions are represented uniformly), the transmitted intensity is obtained by integrating equation (1) :

$$I_a / I = (1/\pi) \int_{-\pi/2}^{\pi/2} \cos^2 \Theta \, d\Theta = 1/2 \qquad (3)$$

Half the intensity is transmitted.

### 1.2. Transposition to photons (quantum mechanics)

How can the above result be interpreted if we move away from wave theory and look at the photons in the beam individually?

The luminous intensity corresponds to the energy transported by a certain number of photons. Sharing of the intensity at the analyser's output can therefore only be due to sharing of the photon flow between the **a** and $\underline{\mathbf{a}}$ channels.

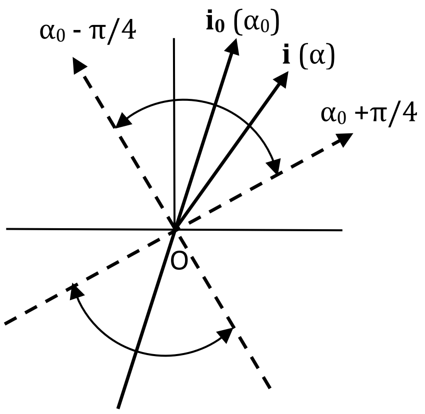
However, since the starting point is a monochromatic beam in a well-defined state of rectilinear polarisation, it seems reasonable to assume that each photon has the same energy and interacts with the analyser in the same way. How then can we explain the division?

This is where quantum theory comes in: polarisation is considered to be a physical quantity with respect to which the photon is in a superposition of states; passing through the analyser constitutes a measurement of polarisation in the orthogonal basis ($\mathbf{a}$, $\underline{\mathbf{a}}$). The result of the measurement is $\mathbf{a}$ or $\underline{\mathbf{a}}$, which means that the photon's polarisation is projected in the direction ($\mathbf{a}$) or in the direction ($\underline{\mathbf{a}}$) with the probabilities given by relations (1) or (2).

### 1.3. A new approach to polarisation

Let's try to imagine an approach in which the photon undergoes no change of state as it passes through the polariser: the polariser simply selects the photons.

1.3.1 **To do this, it must be assumed that the polarisation of the photon, in the direction (i), oscillates around a main direction ($\mathbf{i_0}$).**



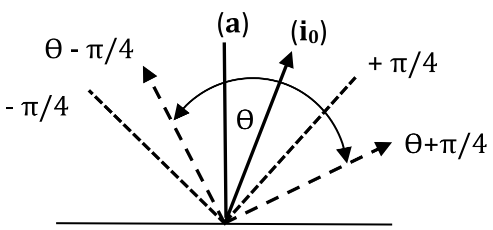

**More precisely, we assume that polarisation oscillates in an angle equal to ± π/4 around ($\mathbf{i_0}$).**

Let us denote by $p(\alpha-\alpha_0)$ the probability density of the presence of ($\mathbf{i}$). We will see later what function to adopt for p to find Malus's law.

*In what follows, we will consider the upper half-plane. However, the polarisation model we are proposing includes a part that is symmetrical with respect to O. We will see in section 1.3.4 that it is necessary to consider ($\mathbf{i_0}$) and ($\mathbf{i}$) as oriented directions.*

1.3.2 First we need to define the photon selection condition for the polariser.

**We assume that a photon is transmitted by a polariser of direction (a) if, when it is about to interact with the device, the direction of polarisation (i) of this photon lies within the angle equal to ± π/4 around (a).**




Take the direction ($\mathbf{a}$) of the polariser as the original axis. Let θ be the angle of the main direction of polarisation of the photon ($\mathbf{i_0}$ ) with **($\mathbf{a}$)** .
With the assumption just made, the photon is transmitted if ($\mathbf{i}$) lies between the directions ($\theta$ - $\pi/4$) and ($\pi/4$) (for $\theta>0$)[4] .

[4] For $\theta < 0$ the photon is transmitted if ($\mathbf{i}$) lies between the directions (- $\pi/4$) and ($\theta + \pi/4$).

Malus's law is satisfied if (for a half-plane) :

$$\int_{\Theta-\pi/4}^{\pi/4} p(\alpha - \Theta)\, d\alpha = (\cos^2 \Theta)/2 \text{ and if : } \int_{-\pi/4}^{\Theta-\pi/4} p(\alpha - \Theta)\, d\alpha = (\sin^2 \Theta)/2$$

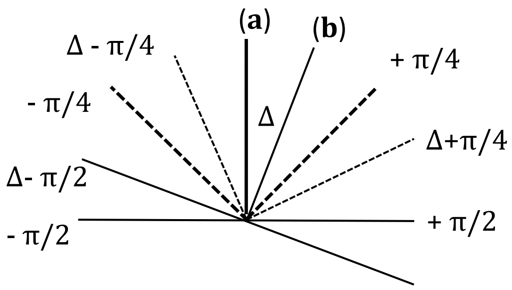
This can be obtained by taking: $p(\alpha) = \cos(2\alpha)/2$.
The probability density is maximal (equal to 0.5) for $\alpha = 0$ and decreases to 0 when $|\alpha|$ varies from 0 to $\pi/4$. The probability of total presence is equal to 1 :

$$2\int_{-\pi/4}^{\pi/4} p(\alpha)\, d\alpha = 1$$

**What is the composition of the photon beam obtained after passing an unpolarised beam through a polariser?** The answer is straightforward: photons from any principal direction of polarisation are transmitted, but with a density in the beam given by $\cos^2 \Theta$. By integrating, we can see that half the photons are transmitted (see equation 3).

**As mentioned at the beginning of 1.3, the polarizer has simply selected the photons; it has not modified the individual polarization state of each photon.**

1.3.3 <u>Let us now apply the result just obtained to the passage of the beam through a second polarizer with a direction (**b**) different from (**a**)</u>. We denote the angle (**a**,**b**) by Δ. In the presentation below, we assume: $0 < \Delta < \pi/4$.




*Let's assume for the moment that this second polariser is placed at such a distance from the first that the direction (**i**) of polarisation of the incoming photons is unchanged from that observed on the first polariser.*

For each range of values of Θ, the table below gives the range of admissible (**i**) and the fraction of the number of Θ photons transmitted by b.

From the fraction of photons Θ transmitted, the fraction transmitted can be calculated for each range as a whole, and then, by summing over the ranges, the fraction of the initial number of (unpolarised) photons that is transmitted by the polariser **b** :

| Range of Θ values | Range of **i**(α) transmitted | Fraction of Θ photons transmitted | Fraction transmitted per range (all Θ) |
|---|---|---|---|
| $-\pi/2 < \Theta < \Delta - \pi/2$ | no | 0 | 0 |
| $\Delta - \pi/2 < \Theta < 0$ | $\Delta - \pi/4 < \alpha < \Theta + \pi/4$ | $(1+\cos(2\Theta - 2\Delta))/4$ | $\pi/8 - \Delta/4 - (\sin 2\Delta)/8$ |
| $0 < \Theta < \Delta$ | $\Delta - \pi/4 < \alpha < \pi/4$ | $(1+\cos(2\Delta))/4$ | $\Delta \cos^2 \Delta/2$ |
| $\Delta < \Theta < \pi/2$ | $\Theta - \pi/4 < \alpha < \pi/4$ | $(\cos(2\Delta) + \cos(2\Theta - 2\Delta))/4$ | $(\pi/8 - \Delta/4)\cos(2\Delta) + (\sin 2\Delta)/8$ |

**Sum**: $\pi(\cos^2 \Delta)/4$

The sum above is to be divided by π since Θ varies from $-\pi/2$ to $\pi/2$.

The total fractions transmitted by polariser **b** is therefore: $(\pi (\cos^2 \Delta) /4)/\pi = (\cos^2 \Delta) /4$.

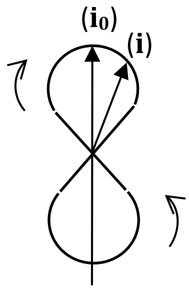
By relating this number to the number of photons that would have passed through the polarizer **a** (i.e. $1/4$ for a half-plane), we obtain the ratio between the number of photons leaving and entering **b** :

$$\text{Ratio } \mathbf{b/a} = \cos^2 \Delta \text{ with } \Delta = \text{angle } (\mathbf{a},\mathbf{b}) \quad (4)$$

We find the result of the quantum prediction. This result can be extended without difficulty to all values of Δ.

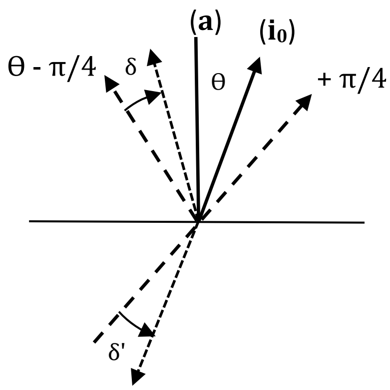
1.3.4 Finally, we need to return to the assumption made at the beginning of the previous paragraph.

*Let's check that the result obtained can be extended to any case where the directions **(i)** of polarisation of the incoming photons are offset from those observed at the output of the first polariser.*




**We will show that the result given by formula (4) is invariant if the direction of polarisation (i) of the photon describes an "8" curve. We are in fact introducing a polarisation vector.**[5]

In this case, the polarisation appears right on one half of the curve and left on the other half. [6]




Let's assume that: $\Theta > 0$. The photons transmitted by the polarizer **a** have a polarization direction (**i**) between the directions $(\Theta - \pi/4)$ and $(\pi/4)$ _ at the instant of their transmission.

What is the effect of a time shift Δt on the position of this angular range when the photons approach the polariser **b**? This time shift corresponds to the angular shifts at the ends of the range, δ and δ', shown above assuming right-polarised photons for $\Delta t > 0$.

These angular offsets are such that :

$$\int_{\Theta-\pi/4}^{\Theta-\pi/4+\delta} p(\alpha - \Theta)\, d\alpha = \int_{3\pi/4}^{3\pi/4+\delta'} p(\alpha - \Theta)\, d\alpha = K\,\Delta t \qquad \text{(K proportionality constant)}$$

The probability of presence in an angular interval is proportional to the time taken to cover the interval.

Therefore : $\int_{\Theta-\pi/4}^{\pi/4} p(\alpha - \Theta)\, d\alpha = \int_{\Theta-\pi/4+\delta}^{\pi/4} p(\alpha - \Theta)\, d\alpha + \int_{3\pi/4}^{3\pi/4+\delta'} p(\alpha - \Theta)\, d\alpha$

[5] See chapter 3: Photon polarisation model. The image of the curve is distorted.

[6] Taking a direction for ($\mathbf{i_0}$ ) means that the polarisation can be described as right or left.

We check that the shifted range ($\Theta$ - $\pi/4 + \delta$, $\pi/4 + \delta'$) does indeed lead to the same fraction of transmitted photons as the initial range.

If we now complete the table in paragraph 1.3.3 to take into account the symmetrical part, we can see that, because of the inversion of the polarisation direction, a shift for one half of the curve is compensated by an opposite shift for the other half. The final sum is unchanged.

*In Chapter 3, we describe a possible polarisation model for the photon that satisfies the above conditions: probability density $p(\alpha) = \cos(2\alpha)/2$ for ($\mathbf{i}$) and a "8" curve with the longitudinal axis as the principal direction ($\mathbf{i_0}$ ).*

## 1.4. Differences between the two approaches

Quantum mechanics, on the one hand, and the approach we have just described, on the other, lead to the same prediction of the results of polarisation experiments. However, there are fundamental differences between the two approaches.

**For Quantum Mechanics** :

- A photon has no predefined polarisation state. The photon can only be assigned a polarisation direction after passing through a polariser. When a polariser selects the ($\mathbf{a}$) orientation, the photon changes to the ($\mathbf{a}$) eigenstate in the polariser base;
- But the result of the passage of a photon of polarisation ($\mathbf{a}$) through a polariser of orientation ($\mathbf{b}$) ≠ ($\mathbf{a}$) is indeterminate. The same experiment repeated on identical photons gives the two possible results (selection or non-selection), for which all we can know in advance is the probability of occurrence.

Indeterminacy is a basic principle of the quantum model: in the orthogonal basis ($\mathbf{b}$, $\underline{\mathbf{b}}$) the photon is not in its own state, but in a superposition of states.

The rule for calculating quantum probabilities can be used to find Malus's law.

**In the proposed alternative approach** :

- The polarisation state of a photon is perfectly determined and independent of the polarisers. It is defined by two variables: ($\mathbf{i_0}$ ) _ main polarisation vector _ and ($\mathbf{i}$) _ polarisation vector at a given instant. This state is not modified by the polarisers, which simply select or reject the photon;
- The result of a photon passing through a polariser, whose directions ($\mathbf{i_0}$ ) and ($\mathbf{i}$) are known, is perfectly predictable. The same experiment repeated on photons with the same polarisation state gives a unique result;

- However, if we consider all the photons selected by a first polariser **a,** passing through a second polariser **b** statistically leads to the same results as those predicted by quantum mechanics.

The probabilization of the results does not reflect a fundamental indeterminacy inherent in the photon, but comes from the dispersion of the polarization directions (**i**) on arrival at the polarizer.

The random nature of the dispersion results from the initial conditions of photon emission. In the polarisation model, the probability density of the presence of the directions (**i**) is chosen so as to reproduce Malus' law.

## 2. Pairs of entangled photons

### 2.1. Quantum entanglement of two photons

Consider a pair of photons forming a system $\{S_1 + S_2\}$. Let's look at the polarisation part of the state vector.

Quantum mechanics admits that this system can have states that cannot be written as the tensor product of a state of the subsystem $S_1$ and a state of the subsystem $S_2$. Such non-separable states are known as intricate states.

Thus, in the Bohm variant of the EPR thought experiment[7], a source emits a pair of photons $\nu_1$ and $\nu_2$ travelling in opposite directions along the Oz axis. It is assumed that the polarisation part (in the Ox, Oy plane) of the state vector describing the photon pair can be written:

$$| \psi(\nu_1, \nu_2) \rangle = (1/\sqrt{2}) \{| \psi(x, x) \rangle + | \psi(y, y) \rangle \}$$

where $| x \rangle$ and $| y \rangle$ are linear polarisation states.

We cannot assign a polarisation to each photon individually. Photons are said to be entangled.

Let us now assume that the photon $\nu_1$ passes through a polariser oriented in the (**a**) direction and the photon $\nu_2$ through a polariser oriented in the (**b**) direction. Let's also assume that the interaction of the photon $\nu_1$ with the polariser **a** occurs first and let's take, for simplicity, the direction (**a**) parallel to the Ox axis.

If the photon $\nu_1$ is projected into the state $| x \rangle$, which occurs with probability equal to 1/2, application of the rules of quantum mechanics leads to the following new state for the two-photon system:

$$| \psi'(\nu_1, \nu_2) \rangle = | x, x \rangle$$

The photon $\nu_2$ is therefore also projected into the $| x \rangle$ state. After passing through the polariser **b**, the result (transmitted photon or not) is given by Malus' law applied with angle (**a**,**b**).

We can therefore say that knowledge of the result of the experiment carried out on the photon $\nu_1$ makes it possible to predict the result of the one carried out on the photon $\nu_2$.

If we denote by $P_{++}(\mathbf{a},\mathbf{b})$, $P_{+-}(\mathbf{a},\mathbf{b})$, $P_{--}(\mathbf{a},\mathbf{b})$ et $P_{-+}(\mathbf{a},\mathbf{b})$the different probabilities of joint detection of $\nu_1$ and $\nu_2$ in the + or - output channels of the polarizers, the predictions of Quantum Mechanics are finally as follows:

$$P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \cos^2(\mathbf{a},\mathbf{b})/2$$
$$P_{+-}(\mathbf{a},\mathbf{b}) = P_{-+}(\mathbf{a},\mathbf{b}) = \sin^2(\mathbf{a},\mathbf{b})/2 \quad (5)$$

[7] cf. Alain Aspect: "Naïve presentation of Bell's inequalities".

### 2.2. Intricate new approach to polarisation

In the new approach described in sub-chapter 1.3, we have two a priori possibilities for defining entanglement: **two photons are polarisation entangled if they have the same main polarisation direction ($\mathbf{i}_0$ ) and identical or opposite variable polarisation vectors ($\mathbf{i}$) at all times.** [8]

These twin photons are indistinguishable in terms of selection by a polariser, whatever its orientation.

Since passing through a polariser does not change the state of the photon, there is no difference between an experiment in which the first photon passes through ($\mathbf{a}$), then the second through ($\mathbf{b}$), or one in which a single photon passes successively through ($\mathbf{a}$) and ($\mathbf{b}$).

The results established in sub-chapter 1.3 show that the new approach makes it possible to recover the equations (5) established in the framework of Quantum Mechanics.

### 2.3. Experiments with entangled photons and Bell's theorem

Four sets of experiments can be carried out[9] , of the type described in 2.1, on pairs of entangled photons, using four polariser orientations: ($\mathbf{a}$), ($\mathbf{b}$), ($\mathbf{a'}$), ($\mathbf{b'}$). The combinations formed are: ($\mathbf{a}$ and $\mathbf{b}$) , ($\mathbf{a}$ and $\mathbf{b'}$), ($\mathbf{a'}$ and $\mathbf{b}$), ($\mathbf{a'}$ and $\mathbf{b'}$).

For each set of results we calculate the four probabilities $P_{++}$ , P $P_{+-, -+}$ and $P_{--}$ . Then, from all these probabilities, we calculate the value of a certain expression S ($\mathbf{a}$, $\mathbf{b}$, $\mathbf{a'}$, $\mathbf{b'}$) which satisfies Bell's theorem[10] , which in this case can be stated as follows:

*The expression S is bounded below and above by two numbers independent of the orientations of the polarizers, in this case:*

$$-2 \leq S \leq 2$$

*if :*

- *the results of the detection of photons by a polarizer can be considered as random;*
- *the separability condition is met: for example, for the experiment with the combination ($\boldsymbol{a}$ and $\boldsymbol{b}$) , detection with $\boldsymbol{a}$ is independent of $\boldsymbol{b}$ and detection with $\boldsymbol{b}$ is independent of $\boldsymbol{a}$.*

Experiments confirm the probabilities given by equations (5). With these values, S does not satisfy Bell's inequality for certain choices of orientations ($\mathbf{a}$, $\mathbf{b}$, $\mathbf{a'}$, $\mathbf{b'}$).

This result is interpreted as verification of the non-local nature of quantum mechanics, reflected in the non-separability of the polarisation states of entangled photons.

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[8] We discuss the choice of one or other possibility in Chapter 3.
[9] cf. Alain Aspect: "Naïve presentation of Bell's inequalities".
[10] J.S. Bell: "On the Einstein - Podolsky - Rosen Paradox".
J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt: "Proposed experiment to test local-hidden variables theories".

In the proposed new approach, the non-separability simply results from the fact that the entangled photons cannot be distinguished by a polariser.

If the photon $\mathbf{v_1}$ is selected by $\mathbf{a}$, the photon $\mathbf{v_2}$ also belongs to the set of photons selectable by $\mathbf{a}$; the selection of $\mathbf{v_2}$ by $\mathbf{b}$ is therefore influenced by $\mathbf{a}$. One of the conditions of Bell's theorem is not satisfied.

With quantum theory, the fundamental indeterminacy of the polarisation state of a photon means that the two photons in the pair have to be linked by the hypothesis of a global state vector in order to escape Bell's inequality.

**Finally, it is possible to propose a classical model of photon polarisation that can explain the experimental results, including those that do not satisfy Bell's inequalities.**

The model is developed in the next chapter.

## 3. Photon polarisation model

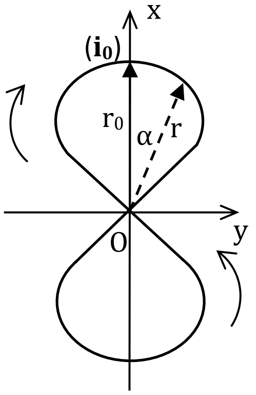
### 3.1. Building the model

3.1.1 Let's start from the conditions set out in paragraph 1.3 :

- *the probability density applying to the polarisation orientation (**i**) is :*

$$p(\alpha) = \cos(2\alpha)/2 \qquad (6)$$

- *polarisation is expressed in vector form and the end of the polarisation vector describes a "8" curve.*




The direction of travel of the curve is defined by the orientation of the main direction of polarisation: by convention, straight polarisation corresponds to the part of the curve encompassing the vector ($\mathbf{i_0}$).

It follows from equation (6) that :

$$(\cos(2\alpha)/2)\, d\alpha = K\, dt \qquad \text{(see 1.3.4)}$$

or, by integrating :

$$\sin(2\alpha)/4 = K\,t + Cte$$

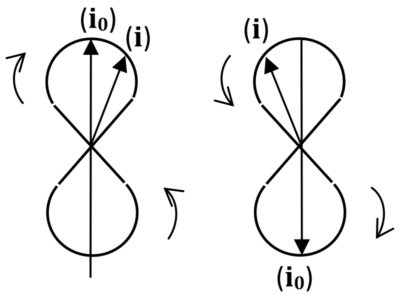
Let's take t =0 for α= 0 (corresponding to the main polarisation orientation ($\mathbf{i_0}$)) and denote by T the period of revolution (corresponding to the complete "8").

We get: $\sin(2\alpha) = 4\,t/T$ (for: $0 \le t < T/4$) (7)

3.1.2 Add the following condition:

- *the superposition of two photons of opposite main polarisation and out of phase by π[11] leads to rectilinear sinusoidal polarisation.*

**We therefore assume that polarisation vectors can be added. This allows us to make a link between photons and wave theory, as we will explain in section 3.2.**




The superposition doubles the component parallel to Ox and cancels out the component parallel to Oy.

We can therefore write :

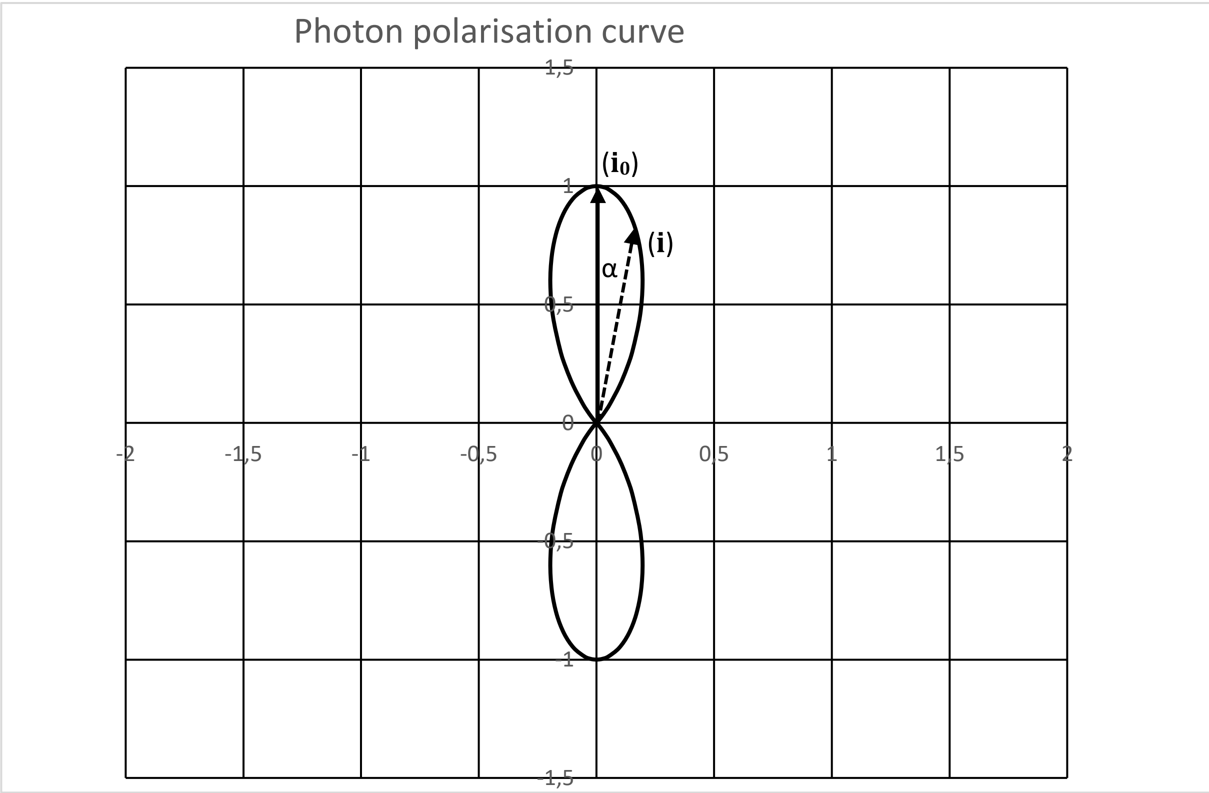
$$r\cos(\alpha) = r_0\cos(\omega t) \text{ where: } \omega = 2\pi/T \qquad (8)$$

$$r\sin(\alpha) = r_0\cos(\omega t)\,\mathrm{tg}(\alpha) \quad \text{with: } \alpha = (\mathrm{Arc}\sin(4t/T))/2 = (\mathrm{Arc}\sin(2\omega t/\pi))/2$$

[11] The phase shift is calculated in relation to the main orientation ($\mathbf{i_0}$ ) of each photon.

Equations (8) above are the parametric equations as a function of time of the curve described by the end of the polarisation vector.

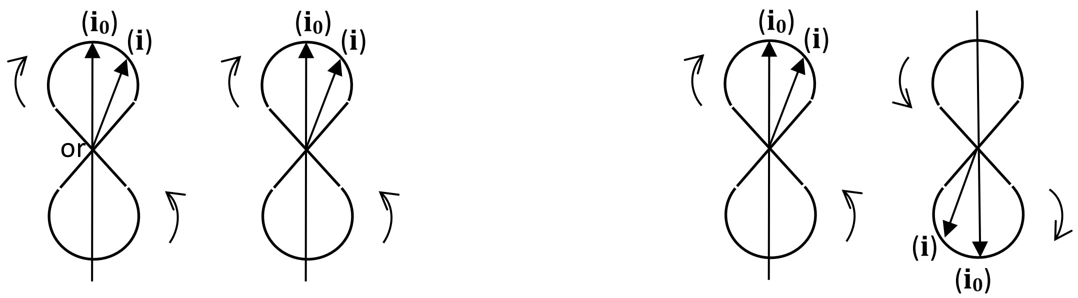
The curve obtained is shown below (with: $r_0 = 1$ and orthonormal axes).




Note that the angular velocity is: $d\alpha/dt = \omega/(\pi \cos(2\alpha)) = 2\nu/\cos(2\alpha)$

3.1.3 <u>Application to entangled photons</u>

The definition of entanglement given in paragraph 2.2 translates as follows:




*photons from ($\boldsymbol{i_0}$) identical in phase*

*photons from ($\boldsymbol{i_0}$) opposite in phase*

We note that the second possibility corresponds, in the case of superposition of the photons, to an absence of polarisation. It would be worth examining whether this could justify choosing this solution.

### 3.2. Link with wave theory

The proposed model attaches a polarisation vector to the photon, which can be analysed as an element of an electric field vector. We have just seen that the superposition of two photons can create an element of rectilinear polarisation.

The transition to wave analysis therefore seems to be possible in continuity with the analysis of individual photons.

However, this presupposes a definition of the way in which photons make up a wave. The flow of photons is organised; in particular, the frequency of the wave defines not only the energy of the photons, but also their succession in time.

This point needs to be analysed further.