Magnetic field

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Link with the electrostatic field

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Summary:

We place ourselves in the weakly relativistic framework (allowing terms of order v^4/c^4 to be neglected in the calculation). From the fundamental principle of dynamics, we establish the equation giving the transform of the force applied to a moving body in a change of Galilean reference frame.

Applied to the problem of electrostatic interaction between two moving charges, this equation gives the expression of the Lorentz force, which introduces a magnetic field in addition to the electrostatic field.

Then, using a geometric approach based on a physical understanding of electrostatic interaction, we show that the same result can be obtained within the framework of classical mechanics.

Taking into account the time shift in the interaction, linked to the distance between the charges, leads to a variation in the rotation of the segment joining them and a variation in their distance, compared with the calculation made when one of the charges is fixed. These variations result in a corrective force to the electrostatic force, which restores the Lorentz force.

The relativistic approach gives an equivalent result because the desynchronisation of the clocks corresponds to a time shift identical to that of the electrostatic interaction. It therefore seems abusive to consider the magnetic field as a consequence of the theory of relativity.

Unlike the electrostatic field, the magnetic field does not involve any exchange of energy with the sources. It is a purely vector field, which should be considered as a tool for correcting a calculation that does not fully model the interaction of charges with each other as they move.

1. Transform of force by change of reference frame

1.1. Force applied to a moving body

The concept of force is associated with the concepts of energy and momentum.

A body with a total energy W has an associated mass m such that: $W = m\,c^2$ (1)
This is the equivalence relationship between mass and energy. [1]

For a body with velocity $\vec{v}$, the momentum is defined as: $\vec{p} = m\,\vec{v}$ (2)

Total energy includes kinetic energy, so it varies with the speed of the body, as does mass.[2]

Consider the derivative of momentum with respect to time :

$$d\vec{p}/dt \;= d(m\,\vec{v})/dt$$

We can write :

$$(d\vec{p}/dt)(\vec{v}\;dt) \;= (\vec{v}\,\vec{v})\,dm + m\,\left(\vec{v}\,\overrightarrow{dv}\right)$$

$$= \;v^2\,dm + m\,v\,dv$$

with: $v = \|\,\vec{v}\,\|$ and : $dv = \|\,\vec{v}\, + \overrightarrow{dv}\,\| - \;\|\,\vec{v}\,\|$

Given (1):

$$(d\vec{p}/dt)(\vec{v}\;dt) = (v^2/c^2)\,dW + \;(v\,dv/c^2) \qquad (3)$$

The fundamental principle of dynamics postulates that $(d\vec{p}/dt)$ represents the force $\vec{F}$ whose work, $\left(\overrightarrow{dp/}dt\right)(\vec{v}\;dt)$, is equal to the variation in energy dW of the body under the action of this force.

So:

$$\vec{F} = \;d\vec{p}/dt \;= d(m\,\vec{v})/dt$$

and, given (3):

$$dW = (v^2/c^2)\,dW + \;(v\,dv/c^2)\,W \qquad (4)$$

Equation (4) can be written as :

$$dW/W \;= (v\,dv/c^2)/(1 - \;v^2/c^2)$$

By integration we obtain: $W = \;\gamma\,W_0 = \;\gamma\,m_0\,c^2$ with: $\gamma = \;1\,/\sqrt{1 - \;v^2/c^2}$ (5)

and therefore: $\vec{p} = \gamma\,m_0\,\vec{v}$ (6)

$\vec{F} = d(\gamma\,m_0\,\vec{v})/dt$ m_0 being the rest mass. (7)

We are going to examine how the force is modified by a change of reference frame.

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[1] Also known as the "Einstein relationship".

[2] It should be remembered that, in our approach to relativity, the term "mass" is not reserved for mass at rest because the latter is not invariant to a change of reference frame

1.2. Change of reference frame

Consider two Galilean reference frames Σ (x, y, z, t) and Σ' (x', y', z', t') in uniform rectilinear relative motion parallel to the axes (x) and (x'). Let us denote by $\vec{u}$ the speed of Σ' relative to Σ.

For an experiment taking place in the Σ reference frame, the equations linking the coordinates in the two reference frames are written :[3]

$$x = x' + u\,t'$$
$$y = y'$$
$$z = z'$$
$$t = t' + u\,x'/c^2 \qquad (8)$$

As a result, the relationship between a velocity $\vec{v}$ in Σ and the corresponding velocity $\vec{v}'$ incan be written in the following simple vector form:

$$\vec{v} = (\overrightarrow{v'} + \vec{u})/(1 + \vec{u}\,\overrightarrow{v'}/c^2) \qquad (9)$$

We will consider the case where the velocities are sufficiently low compared with c that the terms of order v^4/c^4 can be neglected in the calculation. The above equation can be written as:

$$\vec{v} = (\overrightarrow{v'} + \vec{u})(1 - \vec{u}\,\overrightarrow{v'}/c^2) \qquad (9a)$$

We are looking for the relationship between the forces associated with variations in speed $\vec{v}$ and $\overrightarrow{v'}$

$$\vec{F} = m_0\, d(\gamma\,\vec{v})/dt \qquad \text{and} \qquad \overrightarrow{F'} = m'_0\, d(\gamma'\,\overrightarrow{v'})/dt' \qquad (10)$$

a) As the experiment takes place in Σ, you must choose: $m'_0 = \gamma_0\, m_0$
with : $\gamma_0 = 1/\sqrt{1 - u^2/c^2}$

b) From equation (9a), let's express $\overrightarrow{dv}$ as a function of $\overrightarrow{dv'}$:

$$\overrightarrow{dv} = (1 - \vec{u}\,\overrightarrow{v'}/c^2)\overrightarrow{dv'} - (\vec{u}\,\overrightarrow{dv'}/c^2)(\overrightarrow{v'} + \vec{u}) \qquad (11)$$

c) Now let's express $d(\gamma\,\vec{v})$ and $d\left(\gamma'\,\overrightarrow{v'}\right)$:

$$d(\gamma\,\vec{v}) = d\gamma\,\vec{v} + \gamma\,\overrightarrow{dv} = \gamma^3(\vec{v}\,\overrightarrow{dv}/c^2)\,\vec{v} + \gamma\,\overrightarrow{dv}$$

$$d(\gamma\,\vec{v}) = (\vec{v}\,\overrightarrow{dv}/c^2)\,\vec{v} + \gamma\,\overrightarrow{dv}$$

$$d\left(\gamma'\,\overrightarrow{v'}\right) = (\overrightarrow{v'}\,\overrightarrow{dv'}/c^2)\,\overrightarrow{v'} + \gamma'\,\overrightarrow{dv'}$$

[3] Cf. note "*Another approach to relativity*". In this approach, there is no dilation of space and time. The frame of reference in which the experiment takes place is different from the other frames of reference, and the change of coordinates is no longer one-to-one. The coordinate change equations are modified in relation to the Lorentz formulae of special relativity.

d) Let's continue the calculation by expressing $d(\gamma\,\vec{v})$ as a function of $\overrightarrow{v'}$ and $\overrightarrow{dv'}$:

$$d(\gamma\,\vec{v}) = \gamma\,\overrightarrow{dv} + \left(\vec{v}\,\overrightarrow{dv}/c^2\right)\vec{v}$$

$$= \gamma\,(1-\vec{u}\,\overrightarrow{v'}/c^2)\overrightarrow{dv'} - \left(\vec{u}\,\overrightarrow{dv'}/c^2\right)\left(\overrightarrow{v'}+\vec{u}\right) + (\left(\overrightarrow{v'}+\vec{u}\right)\overrightarrow{dv'}/c^2 \quad)\left(\overrightarrow{v'}+\vec{u}\right)$$

$$= \gamma\,(1-\vec{u}\,\overrightarrow{v'}/c^2)\overrightarrow{dv'} + \left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2\right)\left(\overrightarrow{v'}+\vec{u}\right)$$

From relation (9) we can easily derive : $\gamma = \gamma_0\,\gamma'(1+\vec{u}\,\overrightarrow{v'}/c^2)$ which leads to :

$$d(\gamma\,\vec{v}) = \gamma_0\,\gamma'\overrightarrow{dv'} + \left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2\right)\left(\overrightarrow{v'}+\vec{u}\right)$$

e) Now let's add $d\left(\gamma'\,\overrightarrow{v'}\right)$ to the second member

$$d(\gamma\,\vec{v}) = \gamma_0\,(\gamma'\overrightarrow{dv'} + \left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2\right)\overrightarrow{v'}) + \left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2\right)\vec{u}$$

$$d(\gamma\,\vec{v}) = \gamma_0\,d\left(\gamma'\,\overrightarrow{v'}\right) + \left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2\right)\vec{u} \qquad (12)$$

f) Let's move on to the forces by multiplying the two members of equation (12) by m_0/dt :

$$m_0 d(\gamma\,\vec{v})/dt = \gamma_0\,m_0\,d\left(\gamma'\overrightarrow{v'}\right)/dt + m_0\left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2dt\right)\vec{u}$$

Given relations (10), a) and the fact that $dt = (1+\vec{u}\,\overrightarrow{v'}/c^2)\,dt'$, we get :

$$\vec{F} = (1-\vec{u}\,\overrightarrow{v'}/c^2)\overrightarrow{F'} + m_0\left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2dt\right)\vec{u}$$

Since we are neglecting terms of orderv^4/c^4 , we can write :

$$m_0\left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2dt\right) = m'_0\left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2dt'\right) = m'_0\left(\overrightarrow{v'}\,d(\gamma'\overrightarrow{v'})/c^2dt'\right)$$

In other words:

$$m_0\left(\overrightarrow{v'}\,\overrightarrow{dv'}/c^2dt\right) = \overrightarrow{F'}\,\overrightarrow{v'}\,/c^2$$

The equation between the forces in the two reference frames is therefore written :

$$\vec{F} = \left(1-\vec{u}\,\overrightarrow{v'}/c^2\right)\overrightarrow{F'} + (\overrightarrow{F'}\,\overrightarrow{v'}/c^2)\vec{u}$$

$$\vec{F} = \overrightarrow{F'} + (\overrightarrow{F'}\,\overrightarrow{v'}/c^2)\vec{u} - (\vec{u}\,\overrightarrow{v'}/c^2)\overrightarrow{F'}$$

This can be expressed as :

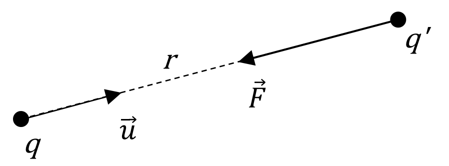
$$\boxed{\vec{F} = \overrightarrow{F'} + \frac{1}{c^2}\overrightarrow{v'}\wedge(\vec{u}\wedge\overrightarrow{F'})} \qquad (13)$$

Remember that this equation is only valid in the weak relativistic case.

2. *Interaction between two moving charges*

2.1. Case of a charge moving in a electrostatic field [4]

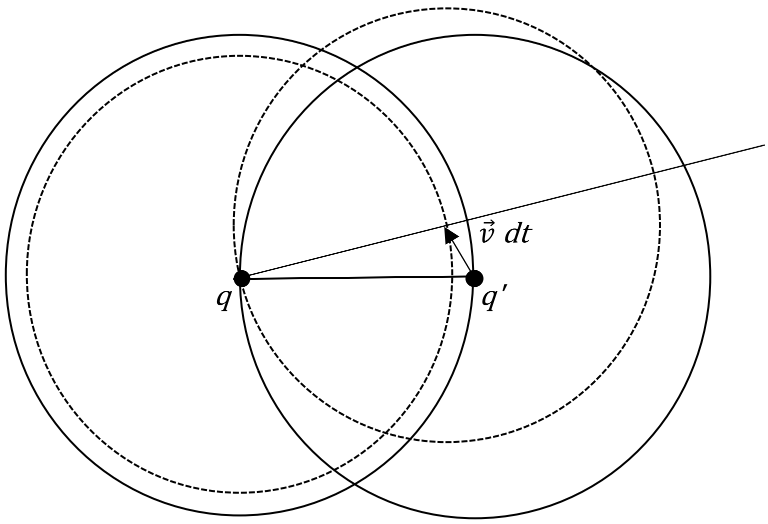
When a charge q' moves in the electrostatic field created by a fixed charge q , even if not radially, its movement is governed by Coulomb's law, which gives the force between the charges :



$$\vec{F} = q' \vec{E}(\vec{r}) = (qq'/ 4\pi \varepsilon_0 r^2) \vec{u}$$

$\vec{E}(\vec{r})$ being the electrostatic field vector.

This is easy to understand if we refer to the energetic description of the interaction, which is analogous to the gravitational interaction.[5]



In the case of a non-radial displacement, the displacement takes place in the plane containing the charges and the velocity vector. A rotation of the segment qq' is added, which simply changes the axis along which the interaction takes place, without changing the energy exchanged.

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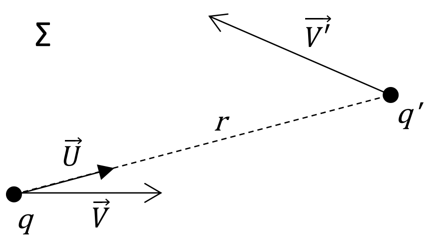
[4] For the construction and properties of the electrostatic field, see note :
"Properties of the electrostatic field. Link with the gravitational field".
[5] cf. note "*Gravitational field, Fundamental Principle of Dynamics and Quantum Mechanics*", paragraph 1.3.2. Interacting sources: energy exchanges between sources and field

2.2. Case of two moving charges

If, in the reference frame in which the experiment takes place, the two charges are mobile (Σ) , can we return to the previous case by a simple change of reference frame (Σ') leading to the immobilisation of one of the charges?

This assumes, at the very least, that the velocity ($\vec{V}$) of one of the charges (q) is constant in the reference frame of the experiment.

In this case, because of the principle of relativity, Coulomb's law can be applied to determine the interaction force in the reference frame Σ' where the charge q is fixed, and then transpose the result to the reference frame Σ using equation (13) obtained in paragraph 1.2.



Let us first observe that the unit vector $\vec{U}$ (carried by the segment qq') is unchanged in the reference frame since there is no contraction of lengths in our approach to relativity.

In Σ', the force acting on the charge q' is therefore :

$$\vec{F'} = (qq'/\, 4\pi\, \varepsilon_0\, r^2)\, \vec{U}$$

On the other hand, the correspondence with the notations in paragraph 1.2 is as follows:

$$\vec{u} \rightarrow \vec{V} \qquad \vec{v} \rightarrow \vec{V'} \qquad \vec{v'} \rightarrow (\vec{V'} - \vec{V})/(1 - \vec{V}\,\vec{V'}/c^2)$$

Equation (13) becomes : $\boxed{\vec{F} = (qq'/\, 4\pi\, \varepsilon_0\, r^2)(\, \vec{U} + \frac{1}{c^2}(\vec{V'} - \vec{V}) \wedge (\vec{V} \wedge \vec{U})\,)}$ (14)

The Lorentz force is expressed as follows:$(\vec{V'} - \vec{V})$ is the velocity of the charge q'relative to the field created by the charge q.$\vec{F}$ is the sum of the electrostatic interaction force and a complementary force:

$$\overrightarrow{\Delta F} = \frac{1}{c^2}(qq'/\, 4\pi\, \varepsilon_0\, r^2)\, ((\vec{V'} - \vec{V}) \wedge (\vec{V} \wedge \vec{U}))$$

Since$\varepsilon_0\, \mu_0 = 1/c^2$, equation (14) can be written as :

$$\vec{F} = q'[(q/\, 4\pi\, \varepsilon_0\, r^2)\, \vec{U} + \left(\vec{V'} - \vec{V}\right) \wedge (\mu_0 q/\, 4\pi\, r^2)(\vec{V} \wedge \vec{U})]$$

or: $\boxed{\vec{F} = q'(\, \vec{E} + \left(\vec{V'} - \vec{V}\right) \wedge \vec{B})}$ (15)

with: $\boxed{\vec{E} = (q/\, 4\pi\, \varepsilon_0\, r^2)\, \vec{U}}$ electrostatic field vector

$\boxed{\vec{B} = (\mu_0 q/\, 4\pi\, r^2)(\vec{V} \wedge \vec{U})}$ magnetic field vector

2.3. Geometric approach

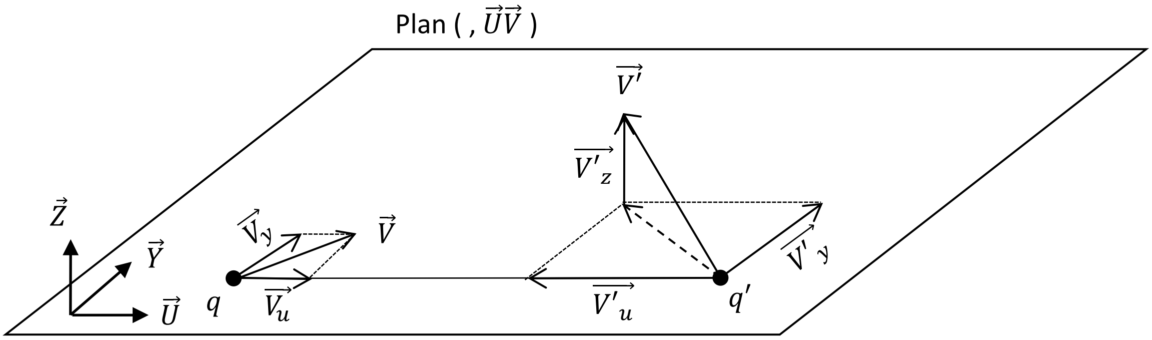
The analysis in paragraph 2.2 derives the appearance of the magnetic field from the application of the theory of relativity.

First of all, the magnetic field is in no way comparable to the electrostatic field or the gravitational field. Unlike the latter, we cannot attribute a physical reality to it, characterised by an energy that is exchanged with the sources.

The magnetic field is a purely vector field, which should be considered as a tool to correct a calculation that does not fully model the interaction of charges with each other as they move.

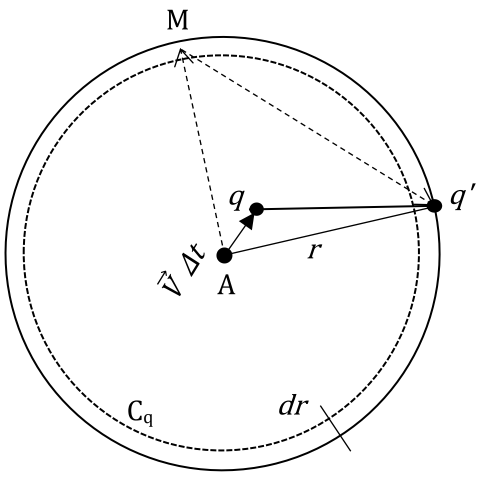
We will show that it is possible to recover equation (14) from a geometric approach, based on a physical understanding of electrostatic interaction, while remaining within the framework of classical mechanics.

($\vec{U}, \vec{Y}, \vec{Z}$) is a local orthonormal reference frame, with $\vec{U}$ directed along qq' and $\vec{Z}$ perpendicular to the plane.



2.3.1. Delayed interaction

Let's look again at the interaction between two charges :[6]



When the distance between the sources varies by dr, the variation in the interaction energy of the source q' is due to a withdrawal of energy from the field created by q , contained in the spherical shell C_q of thickness dr.

The energy captured by q' at point M at time t was emitted by q at time $(t - \Delta t)$, with: . $\Delta t = r/c$ Emission therefore takes place when the load is at a point A such that: $\overrightarrow{Aq} = \vec{V}\Delta t.$

The shift of Δt results in a rotation of the interaction line (from qq' in Σ' to Aq' in Σ).

[6] See footnote [5]

In section 2.3.2, we will see that the variation in rotation during a calculation increment is modified in Σ compared to Σ' , which leads to a corrective term being added to the calculation.

A further correction is necessary because the variation in distance between the loads also differs from one reference frame to another (see paragraph 2.3.3).

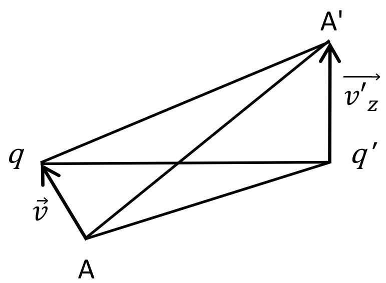
Note: to make the comparison, it is necessary to start from the same distance between the charges at the beginning of the interaction. We simply want to add to the calculation made in Σ' the correction linked to the variations in rotation and distance, all other things being equal. The relativistic calculation does not change the initial distance $\overline{qq'}$ between the two reference frames.

To simplify writing, we reduce the length $\overline{qq'}$ to the unit in Σ', Δt is then reduced to 1/c; the displacement at a speed $\vec{V}$ during this time is $\vec{V}$ /c (which we will note $\vec{v}$).

In view of the above remark , $\overline{A\,q'}$ must also be reduced to 1 in Σ.

On the other hand, the energy captured at M reaches q'after a period of time: .$\Delta t' = \overline{Mq'}/c$ The duration varies with the position of M. The average duration of the energy exchange calculated over the whole shell is equal to $\frac{4}{\pi}\ r/c$. We will not examine the consequences of this time lag, which essentially means that the actions calculated also have to be shifted in time.

Let's start by showing that the $\overrightarrow{V'_z}$ component of $\overrightarrow{V'}$ is irrelevant:



$\overrightarrow{V'_z}$ is orthogonal to the plane($\vec{v}, \overrightarrow{qq'}$).

Since : $\overline{Aq'} = \overline{qq'} = 1$, the two triangles A'qq' and A'Aq' are identical.

There is no change in rotation or distance.

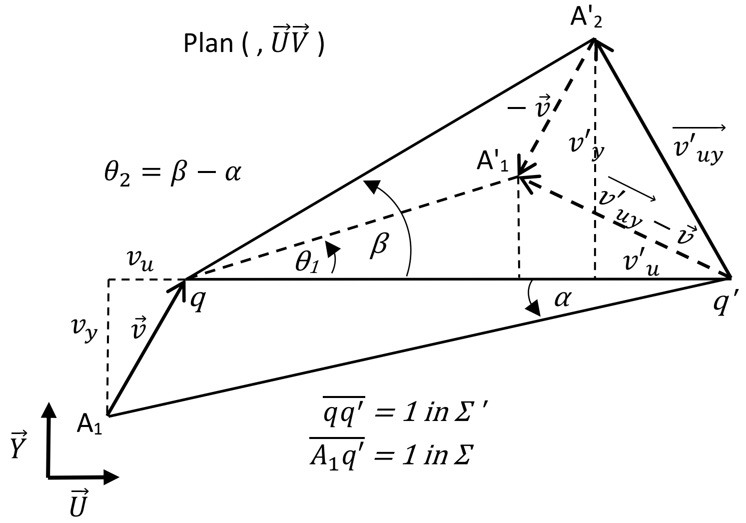
We can therefore limit our calculations to the plane ($\vec{U}$,$\vec{V}$).

2.3.2. Rotation variation

The diagram below shows the rotation of the line of interaction in the two reference frames:

- in Σ (solid lines): $\overrightarrow{A_1q'} \rightarrow \overrightarrow{qA'_2}$ (rotation θ_2) $\overline{A_{1q'}} = 1$ so: $\overline{qq'} = 1 - v_u$
- in Σ' (dotted line): $\overrightarrow{qq'} \rightarrow \overrightarrow{qA'_1}$ (rotation θ_1) $\overline{qq'} = 1$

To obtain the angles at the order v^2 we express the distances according to $\vec{U}$ à *the order v.*



$$\alpha = \frac{v_y}{1} = v_y$$

$$\beta = \frac{v'_y}{1 + v'_u - v_u} = v'_y\,(1 - v'_u + v_u)$$

$$\theta_2 = v'_y\,(1 - v'_u + v_u) - v_y$$

$$\theta_1 = \frac{v'_y - v_y}{1 + v'_u - v_u}$$

$$= (v'_y - v_y)(1 - v'_u + v_u)$$

$(\theta_2 - \theta_1)$ represents the variation in rotation of qq'as seen from the load q. It is its opposite that should be taken for the rotation seen from q' that interests us:

$$\boxed{\Delta_{rotation} = \theta_1 - \theta_2 = v_y\,(v'_u - v_u)} \qquad (16)$$

2.3.3. Distance variation

For this calculation, in order to properly account for all terms of order v^2 , we must express the distance qq'in Σ at this same order :

$$\overline{qq'} = 1 - v_u - {v_y}^2/2 \qquad \text{Let's say : } X = 1 + v'_u - v_u$$

- in Σ : $\overline{qA'_2} = \sqrt{(X - \frac{{v_y}^2}{2})^2 + {v'_y}^2} = X\sqrt{1 - {v_y}^2 + {v'_y}^2} = X - \frac{{v_y}^2}{2} + \frac{{v'_y}^2}{2}$

  $\overline{A_1q'} = 1$

- in Σ' : $\overline{qA'_1} = \sqrt{X^2 + (v'_y - v_y)^2} = X + \frac{1}{2}(v'_y - v_y)^2 = X + \frac{{v_y}^2}{2} + \frac{{v'_y}^2}{2} - v_y v'_y$

  $\overline{qq'} = 1$

- $\Delta_{distance} = (\overline{qA'_2} - \overline{A_1q'}) - (\overline{qA'_1} - \overline{qq'}) = -{v_y}^2 + v_y v'_y$

$$\boxed{\Delta_{distance} = v_y\,(v'_y - v_y)} \qquad (17)$$

Finally, for a time shift equal to r/c, the delayed interaction results in :

- a variation in the rotation of the segment joining the charges :

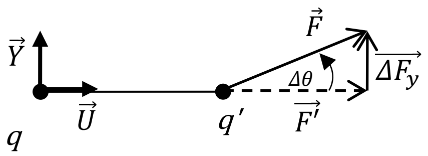
$$\boxed{\Delta_{rotation} = \frac{1}{c^2} V_y\,(V'_u - V_u)} \qquad (18)$$

- a variation in the distance between the charges :

$$\boxed{\Delta_{distance}/r = \frac{1}{c^2} V_y\,(V'_y - V_y)} \qquad (19)$$

2.3.4. <u>Interaction force</u>

a) Corrective force linked to rotation :



The diagram opposite shows that the corrective force $\overrightarrow{\Delta F_y}$ associated with a variation in rotation $\Delta\theta$, orthogonal to $\vec{U}$, is: $\Delta F_y = F'\Delta\theta$.

Therefore, given equation (18):

$$\Delta F_y = \frac{1}{c^2} V_y\,(V'_u - V_u)\,F' \qquad (20)$$

b) Corrective force linked to distance variation :

F' varying in $\frac{1}{r^2}$, we have :

$$dF' = -2\,F'\,\frac{dr}{r}$$

The corrective force $\overrightarrow{\Delta F_u}$, associated with a variation in distance Δr ,

must be calculated incrementally: [7]

$$\int_0^{\Delta t} dr = \frac{1}{\Delta t}\int_0^{\Delta t} \frac{t}{\Delta t}\Delta r\, dt = \frac{\Delta r}{2}$$

As a result:

$$\Delta F_u = -\,F'\,\frac{\Delta r}{r}$$

Given equation (19):

$$\Delta F_u = -\frac{1}{c^2} V_y\left(V'_y - V_y\right) F' \qquad (21)$$

Finally, the force in Σ is written:

$$\vec{F} = \overrightarrow{F'} + \frac{F'}{c^2}\left(V_y\,(V'_u - V_u)\,\vec{Y} - V_y\left(V'_y - V_y\right)\vec{U}\,\right) \qquad (22)$$

which takes the form:

$$\vec{F} = F'(\vec{U} + \frac{1}{c^2}(\overrightarrow{V'} - \vec{V}) \wedge (\vec{V} \wedge \vec{U})) \qquad (22a)$$

This is equation (14).

[7] See note "*Properties of the electrostatic field. Link with the gravitational field*" paragraph 2.1.
The time increments to be considered are equal to the refresh period of the field

3. Comparison of the two approaches to the problem

We have just seen that the relativity approach and the geometric approach lead to the same equation (14) or (22a). How can we explain this?

3.1. Analysis of the relativity approach

The corrective force is generated from equation (12), which is written as follows, substituting the notations used in paragraphs 2.2 and 2.3:

$$d(\gamma\,\vec{V'}) = \gamma_0\, d\left(\gamma'\,(\vec{V'} - \vec{V})\right) + \left((\vec{V'} - \vec{V})\, d(\vec{V'} - \vec{V})/c^2\right)\vec{V}$$

Each term of the second member makes a contribution to the corrective force :

- The first leads to: $\overrightarrow{\Delta F_1} = -(\vec{V}\,(\vec{V'} - \vec{V})/c^2)\vec{F'}$

or : $\overrightarrow{\Delta F_1} = -\frac{F'}{c^2}\left(V_u(V'_u - V_u) + V_y(V'_y - V_y)\right)\vec{U}$

It is induced by the desynchronisation of time between the two reference frames (equation (8)).

- The second gives : $\overrightarrow{\Delta F_2} = (\vec{F'}\,(\vec{V'} - \vec{V})/c^2)\,\vec{V}$

or, since $\vec{F'} = F'\vec{U}$: $\overrightarrow{\Delta F_2} = \frac{F'}{c^2}(V_u(V'_u - V_u)\,\vec{U} + V_y(V'_u - V_u)\vec{Y}\,)$

This second term is derived from equation : $\gamma = \gamma_0\,\gamma'(1 + \vec{V}(\vec{V'} - \vec{V})\,/c^2)$

This equation is simply the relationship between the total energies in the two reference frames, which is a consequence of the law of composition of velocities[8] , and therefore also of the desynchronisation of time.

It is not easy to give a physical interpretation to each part, $\overrightarrow{\Delta F_1}$ and $\overrightarrow{\Delta F_2}$, of the corrective force.

Due to the deletion of two opposite terms, the sum of $\overrightarrow{\Delta F_1}$ and $\overrightarrow{\Delta F_2}$ reduces to :

$$\overrightarrow{\Delta F} = \frac{F'}{c^2}\left(-V_y(V'_y - V_y)\vec{U} + V_y(V'_u - V_u)\vec{Y}\right)$$

This equation is equation (22) of the geometric approach.

[8] Cf. note "*Another approach to relativity*", chapter 3. Relativistic approach based on the equivalence between mass and energy.

As we have seen, this approach starts by taking into account the delayed interaction between the charges, which induces a difference in rotation of the segment connecting them and in distance. The physical interpretation is obvious.

3.2. Explanation of the equivalence of the two approaches

How can we explain that the effect of the desynchronisation of clocks (relativity approach) is equivalent to the effect of the delayed interaction (geometric approach)?

In the note entitled "*Mass, energy and reference frames*" we propose an explanation of the phenomenon of desynchronisation of clocks observed between two reference frames in relative motion, inspired by Mach's principle: [9]

Desynchronisation reflects the time shift in the transmission of the gravitational interaction with the masses of the universe.

Since the refreshment waves of the gravitational field and the electrostatic field have the same speed c, the offset corresponding to the distance $\overline{qq'}$ between the charges is the same.

In conclusion, it would appear to be abusive to consider the magnetic field as a consequence of the theory of relativity. All we can say is that the equations for changing coordinates induced by this theory allow us to obtain the correct evaluation of the field.

[9] See paragraph 4, reproduced in the appendix to this note.

Appendix: Extract from the "*Mass, energy and reference systems*" note

4. Explanation of clock desynchronisation

The desynchronisation of clocks between two frames of reference in relative motion results in observers seeing a distorted image of the other frame of reference at the same instant in one frame, since the clocks in the other frame mark different times depending on their position:

According to equations *(3)*: at time t in Σ: $[x,t] \rightarrow [x' = x - ut',\ t' = t - ux'/c^2]$

at time t' in Σ': $[x',t'] \rightarrow [x_1 = x' + ut_1,\ t_1 = t' + ux_1/c^2]$

We have seen that, in the context of the theory of special relativity, $[x',t']$ gives back $[x,t]$ because of the effect of the deformation of space and time, in addition to the effect of desynchronisation.

The desynchronisation of clocks is linked to the existence of a speed limit.[10] This is the speed of displacement of energy c : the speed of gravitational waves[11] or electromagnetic waves, which is identical in all frames of reference.

So how can we physically explain the phenomenon of desynchronisation?

In a Galilean frame of reference, the operation of synchronising the clocks is based on the fact that, by definition of such a frame of reference, two identical experiments, offset by translation and rotation in space, take place with the same durations. Conversely, we could say that it is the fact of synchronising the clocks so as to assign the same duration to the experiments that makes the reference frame Galilean.

Ernst Mach hypothesised that the inertia of material objects would be induced by all the other masses in the universe

As a consequence of this hypothesis, the Galilean reference frames, considered, as we have seen, as massive "frames of reference", appear in some way to be "in equilibrium" with the rest of the universe, i.e. immobile in relation to the frame constituted by all the masses of this universe.[12]

The question then arises: how can two Galilean reference frames in relative motion both satisfy this condition?

Our answer is as follows: this is only acceptable if time cannot be considered as absolute; we must observe a time shift from one frame of reference to another.

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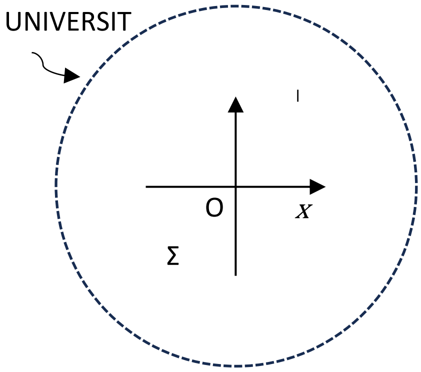
[10] Cf. note "*Another approach to relativity*" paragraph 2.3.

[11] As defined in the note "*Gravitational field, Fundamental Principle of Dynamics and Quantum Mechanics*" paragraph 1.1.3.

[12] The Wikipedia article on *Mach's Principle* states that local inertial reference frames are affected by cosmic motion and the distribution of matter.
Julian Barbour and Herbert Pfiste (eds.), "*Mach's principle: from Newton's bucket to quantum gravity".* (proceedings of the conference held in Tübingen in July 1993), Boston, Basel and Berlin, Birkhäuser , coll. "Einstein studies" (n° 6), August 1995, 1re ed.



In a Galilean frame of reference, interaction with the masses of the universe results in exchanges of energy at speed c (via field refreshment waves).

For a point of abscissa x, the transmission of interactions along the Ox axis is shifted by a time $T = x/c$ relative to the origin O of the reference frame.

Let's now consider the reference frames Σ, assumed fixed, and Σ' moving at speed u along the Ox axis.

If time was absolute, the transmission offset along Ox' in Σ' would be modified in Σ by the value: $\Delta T' = uT'/c = ux'/c^2$

Since Σ' is Galilean, this variation in the offset must be compensated for by desynchronising the clocks of Σ' with respect to Σ.

We find the expected relationship : $t' = t - ux'/c^2$

This result remains true if we consider interaction in any direction. If we denote by α the angle of this direction with respect to Ox', the shift is written :

$$\Delta T' = u\,(x' \cos\alpha/c)\,(u/c \cos\alpha) = ux'/c^2$$